The two-body open charm decays of $Z^+(4430)$

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The two-body open charm decays $Z^+(4430) \to D^+\bar{D}^{*0}, D^{*+}\bar{D}^0, D^{*+}\bar{D}^{*0}$ occur through the re-scattering mechanism and their branching ratios are strongly suppressed if $Z^+(4430)$ is a $D_1\bar{D}^*$ molecular state. In contrast, $Z^+(4430)$ falls apart into these modes easily with large phase space and they become the main decay modes if $Z^+(4430)$ is a tetraquark state. Experimental search of these two-body open charm modes and the hidden charm mode $\chi_{cJ}\rho$ will help distinguish different theoretical schemes.

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I. INTRODUCTION

Belle Collaboration observed a new resonance $Z^+(4430)$ in the invariant mass spectrum of $\psi'\pi^+$ of $B \to K\psi'\pi^+$ [1]. Its mass and width are $m = 4433 \pm 4(\mathrm{stat}) \pm 1(\mathrm{syst})$ MeV and $\Gamma = 44^{+17}_{-13}(\mathrm{stat})^{+30}_{-11}(\mathrm{syst})$ MeV respectively. This charged state stimulated extensive studies of its nature [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Theoretical explanations of $Z^+(4430)$ include the S-wave threshold effect [2], the $D_1(D_1')\bar{D}^*$ molecular state [3, 4, 5, 6, 7], the tetraquark state [8, 9, 10], the cusp effect [11] and the $\Lambda_c - \Sigma_c^0$ bound state [13].

Recently we studied whether $Z^+(4430)$ could be an S-wave D_1D^* ($D_1'D^*$) molecular state considering both the pion and sigma meson exchange potentials [5]. Our numerical results show that there may exist an S-wave $D^*\bar{D}_1$ molecular state with $J^P = 0^-$. If one ignores the width of D_1', there may also exist an S-wave $D^*\bar{D}_1'$ molecular state with $J^P = 0^-, 1^-, 2^-$. However, D_1' will rapidly decay into $D^*\pi$ before the formation of the $D_1'D^*$ bound state because of its large width around 384 MeV.

In this work we assume $Z^+(4430)$ is a $D^*\bar{D}_1$ molecular state with $J^P = 0^-$ and study its two-body open charm decay modes, which may be used to distinguish the tetraquark and molecule picture. This work is organized as follows. After introduction, we discuss the hidden *vs* open charm decays of $Z^+(4430)$. In Section III, the decay amplitude of the two-body open charm decay of $Z^+(4430)$ is given. The last section is the numerical results and discussion.

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II. THE HIDDEN *VS* OPEN CHARM DECAYS OF $Z^+(4430)$

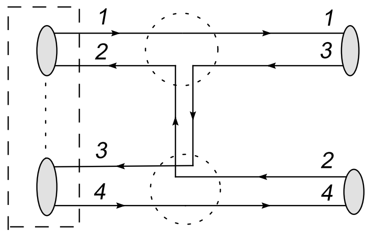
Assuming $Z^+(4430)$ is an S-wave $D^*\bar{D}_1$ molecular state, we illustrate its strong decays allowed by Okubo-Zweig-Iizuka (OZI) rule in Fig. 1. Here indices 1, 2, 3, and 4 are c, $\bar{u}$, $\bar{c}$ and d quarks respectively. The dashed-line box denotes the S-wave $D^* - \bar{D}_1$ molecular state.

A. Hidden charm decays

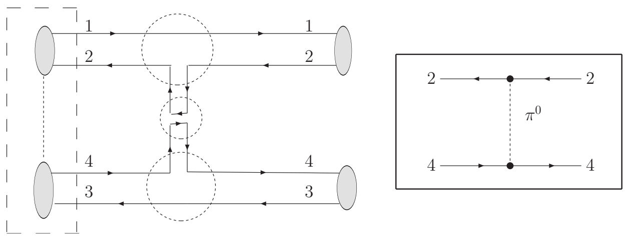
Fig. 1 (a) describes the hidden charm decay process $Z^+(4430) \to D^*\bar{D}_1 \to (c\bar{c})(d\bar{u})$ by exchanging one charmed meson between D^* and $\bar{D}_1$. $Z^+(4430)$ was observed in the $\psi'\pi^+$ channel. If it's the S-wave $D^*\bar{D}_1$ molecular state, its quantum number is $I^G(J^P) = 1^+(0^-, 1^-, 2^-)$ [4]. Kinematically allowed hidden charm decays can be classified into (1) P-wave modes: $\psi\pi$, $\psi'\pi$, $\psi(3S)\pi$, $\psi(4S)\pi$, $\psi(1D)\pi$, $\psi(2D)\pi$, $\eta_c\rho$, $\eta_c(2S)\rho$; (2) S-wave modes: $\chi_{cJ}\rho$. The neutral partner of $Z^+(4430)$ may also decay into $\psi\omega$ via P-wave and $\chi_{cJ}\eta$ via S-wave.

$Z^+(4430)$ was observed only in the $\psi'\pi^+$ channel up to now, which is very puzzling. One of the possible reasons is the mismatch of the Q-values of the initial and final states [4]. If so, one should also expect a signal in the $\pi^+\psi(3S)$ channel since there is nearly no mismatch of the Q-value. Another potential reason is the specific nodal structure in the wave functions of the final states, which was discussed by Bugg recently [12]. Meng and Chao adopted the re-scattering mechanism to explain the suppression of the π^+J/ψ decay mode [3].

Besides the discovery mode $\psi'\pi^+$, we want to emphasize that the S-wave $\chi_{cJ}\rho^+$ mode is very important since there is no additional suppression factor $(\frac{k}{m_Z})^2$ where k is the decay momentum and m_Z is the mass of $Z^+(4430)$. Naively one may expect that the orbital excitation in the D_1 meson easily transfers to χ_{cJ}. Moreover, the $\chi_{c0}\rho^+$



(a)



(b)

FIG. 1: Diagrams (a) and (b) depict the hidden and open charm decay of $Z^+(4430)$ at the quark level respectively . Here 1, 2, 3, and 4 denote c, $\bar{u}$, $\bar{c}$ and d quarks respectively .

mode may help distinguish the J^P of $Z^+(4430)$ and test the $D^*\bar{D}_1$ molecular state assignment. If the $\chi_{c0}\rho^+$ mode is observed, one can exclude the $J^P = 0^-$ possibility for the $D^*\bar{D}_1$ system. We strongly urge experimental colleagues to search $Z^+(4430)$ in the $\chi_{cJ}\pi^+\pi^0$ channel.

B. Open charm decays

Both $Z^+(4430)$ and D_1 mesons have finite widths. $Z^+(4430)$ can decay through the upper tail of its mass distribution into $D_1\bar{D}^*$, especially the lower part of the D_1 mass distribution. As pointed out in Ref. [3], the dominant decay mode of $Z^+(4430)$ is $Z^+(4430) \to D_1\bar{D}^* \to D^*\bar{D}^*\pi$.

Besides the above dominant mode, there exists another class of open charm decay mode through the re-scattering mechanism [17, 18], which is shown in Fig. 1 (b). One pion or ρ (ω) meson exchange occurs between the light quarks. These two-body decay modes include $D\bar{D}^* + D^*\bar{D}$ and $D^*\bar{D}^*$. The $D\bar{D}$ decay mode is forbidden for a pseudoscalar meson.

This kind of open charm decay is strongly suppressed if $Z(4430)$ is a molecular state, as will be shown in the following sections. However, $Z(4430)$ will fall apart into these two-body final states very easily with large phase space if $Z(4430)$ is a tetraquark state as proposed in Refs. [8, 9, 10]. Therefore, these two-body decay modes can be used to distinguish the molecular and tetraquark pictures. This is the key motivation of the present work.

III. THE AMPLITUDE OF THE TWO-BODY OPEN CHARM DECAY

Assuming $Z^+(4430)$ is an S-wave $D^* - \bar{D}_1$ molecular state with $J^P = 0^-$ [5], we derive the two-body open charm decay amplitude using the optical theorem. From the unitarity of the S-matrix $S^\dagger S = 1$, one obtains

$$-i(T - T^\dagger) = T^\dagger T \tag{1}$$

where $S = 1 + iT$. We sandwich the above equation between the one-particle state $|P\rangle$ and two-particle state $|k_1, k_2\rangle$. By inserting a complete set of intermediate states to the right-hand side, one gets

$$\begin{aligned}&\langle k_1, k_2|T^\dagger T|P\rangle \\ =\ &\sum_\alpha\Big(\prod_{i=1}^{\alpha}\int\frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\Big)\langle k_1, k_2|T^\dagger|p_i\rangle\langle p_i|T|P\rangle.\end{aligned} \tag{2}$$

Now the absorptive part reads

$$\begin{aligned}&\texttt{Abs}\,\mathcal{M}(A(P) \to B(k_1)C(k_2)) \\ =\ &\frac{1}{2}\sum_\alpha\Big(\prod_{i=1}^{\alpha}\int\frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\Big)(2\pi)^4\delta^4(k_1 + k_2 - \sum_{i=1}^{\alpha}p_i) \\ &\times\mathcal{M}[k_1k_2 \to \{p_i\}]\mathcal{M}^*[P \to \{p_i\}].\end{aligned} \tag{3}$$

The optical theorem relates the absorptive part of the $A \to B + C$ decay amplitude to the sum of all possible $A \to \{p_i\}$ decays and the re-scattering process $\{p_i\} \to B + C$.

Thus, for the two-body charm decay $Z^+(4430) \to \bar{D}_1D^* \to D^{(*)}\bar{D}^{(*)}$, the absorptive part of the decay amplitude can be written as

$$\begin{aligned}&\texttt{Abs}[Z^+(4430) \to \bar{D}_1D^* \to D^{(*)}\bar{D}^{(*)}] \\ =\ &\frac{|\mathbf{p}|}{32\pi^2M_Z}\int d\Omega\mathcal{M}^*[Z^+ \to D^*\bar{D}_1] \\ &\times\mathcal{M}[D^*\bar{D}_1 \to D^{(*)}\bar{D}^{(*)}]\end{aligned} \tag{4}$$

where $|\mathbf{p}|$ denotes the three momentum of intermediate state in the center of mass frame of $Z^+(4430)$.

Because the mass of $Z^+(4430)$ is close to the sum of the masses of D_1 and D^*, the dispersive part of $Z^+(4430) \to \bar{D}_1D^* \to D^{(*)}\bar{D}^{(*)}$ amplitude is very important as shown in the case of X(3872) by Meng and Chao [18]. The dispersive part of the amplitude is related to the absorptive part through the dispersion relation,

$$\texttt{Dis}\mathcal{M}(M_Z) = \frac{1}{\pi}\int_{s_0^2}^{\infty}\frac{\texttt{Abs}\mathcal{M}(s)}{s - M_Z^2}ds, \tag{5}$$

with

$$\begin{aligned}\texttt{Abs}\mathcal{M}(s)\ &=\ \texttt{Abs}[Z^+(4430) \to \bar{D}_1D^* \to D^{(*)}\bar{D}^{(*)}] \\ &\quad\times\exp(-\eta|\mathbf{k}|^2),\end{aligned} \tag{6}$$

where $|\mathbf{k}| = [\lambda(M_Z^2, m_{D^*}^2, m_{D_1}^2)]^{1/2}/2M_Z$ is the three momentum of the intermediate state in the rest frame of

$Z^+(4430)$. $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$ is källen function. One chooses $s_0 = m_{D_1}+m_{D^*}$. The exponential not only describes the dependence of the interaction between $Z^+(4430)$ and $D^*\bar{D}_1$ on $|\mathbf{k}|$, but also plays the role of the cutoff. The factor η is related to the interaction radius R by $\eta = R/6$ [19].

In order to get $\mathcal{M}^*[Z^+ \to D^*\bar{D}_1]$ and $\mathcal{M}[D^*\bar{D}_1 \to D^{(*)}D^{(*)}]$, we use the following Lagrangians [3, 20]

$$\mathcal{L}_{Z^+D_1D^*} = g_{Z^+D_1D^*}Z(D^*\cdot D_1^\dagger)+h.c., \qquad (7)$$

$$\begin{aligned}\mathcal{L}_1 \;=\; & \frac{1}{2}g_{\mathcal{D}^*\mathcal{D}^*\mathbb{P}}\varepsilon_{\mu\nu\alpha\beta}\mathcal{D}^{*\mu}_{i}\partial^\nu\mathbb{P}^{ij}\overleftrightarrow{\partial}^{\alpha}\mathcal{D}^{*\beta\dagger}_{j} \\ & -ig_{\mathcal{D}^*\mathcal{D}\mathbb{P}}(\mathcal{D}^i\partial^\mu\mathbb{P}_{ij}\mathcal{D}^{*j\dagger}_{\mu}-\mathcal{D}^{*i}_{\mu}\partial^\mu\mathbb{P}_{ij}\mathcal{D}^{j\dagger}) \\ & -ig_{\mathcal{D}\mathcal{D}\mathbb{V}}\mathcal{D}^\dagger_i\overleftrightarrow{\partial}_\mu\mathcal{D}^j(\mathbb{V}^\mu)^i_j \\ & -2f_{\mathcal{D}^*\mathcal{D}\mathbb{V}}\varepsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathbb{V}^\nu)^i_j(\mathcal{D}^\dagger_i\overleftrightarrow{\partial}^{\alpha}\mathcal{D}^{*\beta j}-\mathcal{D}^{*\beta\dagger}_{i}\overleftrightarrow{\partial}^{\alpha}\mathcal{D}^j) \\ & +ig_{\mathcal{D}^*\mathcal{D}^*\mathbb{V}}\mathcal{D}^{*\nu\dagger}_{i}\overleftrightarrow{\partial}_\mu\mathcal{D}^{*j}_{\nu}(\mathbb{V}^\mu)^i_j \\ & +4if_{\mathcal{D}^*\mathcal{D}^*\mathbb{V}}\mathcal{D}^{*\dagger}_{i\mu}(\partial^\mu\mathbb{V}^\nu-\partial^\nu\mathbb{V}^\mu)^i_j\mathcal{D}^{*j}_{\nu}, \end{aligned} \qquad (8)$$

$$\begin{aligned}\mathcal{L}_{D_1D^*\pi} \;=\; & ig_{D_1D^*\pi}[-3D^*_\mu D^\dagger_{1\nu}\partial^\mu\partial^\nu\pi+(D^*\cdot D_1^\dagger)\partial^2\pi \\ & -\frac{1}{m_{D^*}m_{D_1}}\partial_\mu D^{*\rho}\partial_\nu D^\dagger_{1\rho}\partial^\mu\partial^\nu\pi]+h.c., \end{aligned} \qquad (9)$$

where $\mathcal{D}$ and $\mathcal{D}^*$ are pseudoscalar and vector heavy mesons respectively, i.e. $\mathcal{D}^{(*)}$=$((\bar{D}^0)^{(*)}$, $(D^-)^{(*)}$, $(D_s^-)^{(*)})$. $\mathbb{P}$ and $\mathbb{V}$ denote the octet pseudoscalar and the nonet vector meson matrices. The values of the coupling constants will be given in Section IV. The Lagrangian relevant to the interaction of D_1 with $\rho(\omega)$ and $D^{(*)}$ mesons is given in Ref. [21].

The flavor wave function of $Z^+(4430)$ is [4]

$$|Z^+(4430)\rangle = \frac{1}{\sqrt{2}}\Big(|D^{*+}\bar{D}_1{}^0\rangle + |\bar{D}^{*0}D_1{}^+\rangle\Big)\,. \qquad (10)$$

Thus $Z^+(4430)$ with $J^P = 0^-$ can decay into $D^{*+}\bar{D}^0$, $D^+\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*0}$ via the intermediate states $D^{*+}\bar{D}_1{}^0$ and $\bar{D}^{*0}D_1{}^+$. If one ignores the mass difference between neutral and charged charmed mesons, the contribution from the intermediate state $D^{*+}\bar{D}_1{}^0$ is same as that from $\bar{D}^{*0}D_1{}^+$.

A. $Z^+(4430) \to D^{*+}\bar{D}_1{}^0(\bar{D}^{*0}D_1{}^+) \to \bar{D}^{*0}D^+$

Fig. 2 (b) describes the open charm decay $Z^+(4430) \to D^{*+}\bar{D}_1{}^0 \to \bar{D}^{*0}D^+$. The absorptive part of the decay amplitude of $Z^+(4430) \to \bar{D}_1^0(p_1,\epsilon_1)D^{*+}(p_2,\epsilon_2) \to$

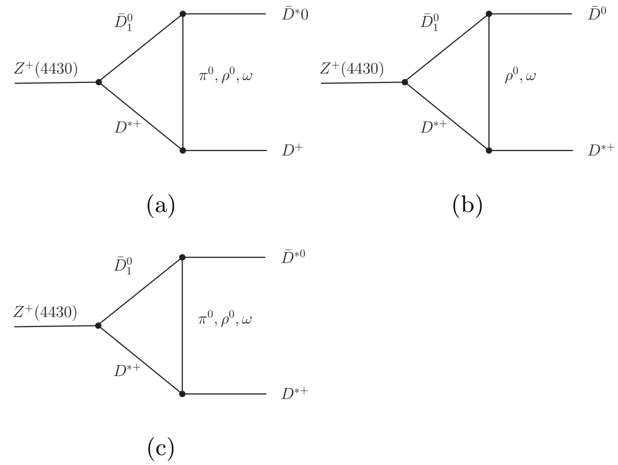


FIG. 2: The two-body open charm decays of $Z^+(4430)$.

$\bar{D}^{*0}(p_3,\epsilon_3)D^+(p_4)$ via the π meson exchange is

$$\begin{aligned} & \mathtt{Abs}^{\pi}[Z^+ \to D^{*+}\bar{D}_1{}^0 \to D^+\bar{D}^{*0}] \\ =\; & \frac{|\mathbf{p}|}{32\pi^2 M_Z}\int d\Omega\; i[ig_{Z^+D_1D^*}] \\ & \times[i(-i)\frac{g_{D^*DP}}{\sqrt{2}}(iq^\mu)]ig_{D_1D^*\pi}[-3q^\beta q^\nu \\ & +g^{\beta\nu}q^2-\frac{g^{\nu\beta}}{m_{D^*}m_{D_1}}(p_3\cdot q)(p_1\cdot q)]\epsilon_{3\beta} \\ & \times\Big[-g_{\nu\rho}+\frac{p_{1\nu}p_{1\rho}}{m^2_{D_1}}\Big]\Big[-g^\rho_\mu+\frac{p_2^\rho p_{2\mu}}{m^2_{D^*}}\Big] \\ & \times\frac{\mathcal{F}^2(q^2,m_i^2)}{q^2-m_\pi^2}. \end{aligned} \qquad (11)$$

The amplitude from the $\rho(\omega)$ exchange is

$$\begin{aligned} & \mathtt{Abs}^{\rho(\omega)}[Z^+ \to D^{*+}\bar{D}_1{}^0 \to D^+\bar{D}^{*0}] \\ =\; & \frac{|\mathbf{p}|}{32\pi^2 M_Z}\int d\Omega\; i[ig_{Z^+D_1D^*}] \\ & \times[-2I_1^{(i)}if_{D^*DV}\varepsilon_{\mu\nu\alpha\beta}(iq^\mu)(-ip_4^\alpha-ip_2^\alpha)] \\ & \times I_2^{(i)}[i\varepsilon_{\tau\sigma\xi\kappa}\frac{p_1^\kappa}{m_{D_1}}\epsilon_3^\xi\Theta^\sigma_\eta(g'_{2,2}+g'_{1,2}) \\ & +i\varepsilon_{\tau\sigma\eta\kappa}\frac{p_1^\kappa}{m_{D_1}}\epsilon_{3\gamma}\Theta^{\sigma\gamma}(g'_{2,2}-g'_{1,2})+i\varepsilon_{\tau\eta\xi\kappa}\epsilon_3^\xi\frac{p_1^\kappa}{m_{D_1}}g'_{1,0}] \\ & \times\Big[-g^{\tau\nu}+\frac{q^\tau q^\nu}{m_i^2}\Big]\Big[-g^\eta_\lambda+\frac{p_1^\eta p_{1\lambda}}{m^2_{D_1}}\Big] \\ & \times\Big[-g^{\beta\lambda}+\frac{p_2^\beta p_2^\lambda}{m^2_{D^*}}\Big]\frac{\mathcal{F}^2(q^2,m_i^2)}{q^2-m_i^2} \end{aligned} \qquad (12)$$

with

$$\begin{aligned} & \Theta^{ab} \\ =\; & \frac{g^{ab}(q\cdot p_1)^2}{3m^2_{D_1}}+\frac{2p_1^a p_1^b(q\cdot p_1)^2}{3m^4_{D_1}}-\frac{p_1^a q^b(q\cdot p_1)}{m^2_{D_1}} \\ & -\frac{p_1^b q^a(q\cdot p_1)}{m^2_{D_1}}+q^a q^b-\frac{g^{ab}q^2}{3}+\frac{p_1^a p_1^b q^2}{3m^2_{D_1}}, \end{aligned} \qquad (13)$$

where the index i in $I_{1,2}^{(i)}$ and m_i denotes the $\rho(\omega)$ meson. $I_1^{(\rho)} = -I_2^{(\omega)} = -\frac{1}{\sqrt{2}}$ and $I_2^{(\rho)} = I_2^{(\omega)} = \frac{1}{\sqrt{2}}$. The same notations are also used in the following subsections.

B. $Z^+(4430) \to D^{*+}\bar{D}_1{}^0(\bar{D}^{*0}D_1{}^+) \to \bar{D}^0 D^{*+}$

The open charm decay $Z^+(4430) \to D^{*+}\bar{D}_1{}^0 \to \bar{D}^0 D^{*+}$ is depicted in Fig. 2 (c). The absorptive part of the decay amplitude of $Z^+(4430) \to \bar{D}_1^0(p_1,\epsilon_1)D^{*+}(p_2,\epsilon_2) \to \bar{D}^0(p_3)D^{*+}(p_4,\epsilon_4)$ from the $\rho(\omega)$ meson exchange is

$$\begin{aligned}
&\texttt{Abs}^{\rho(\omega)}[Z^+ \to D^{*+}\bar{D}_1{}^0 \to D^{*+}\bar{D}^0]\\
=& \frac{|\mathbf{p}|}{32\pi^2 M_Z}\int d\Omega\ i[ig_{Z^+D_1D^*}]\\
&\times I_1^{(i)}[-2ig_{D^*D^*V}p_{2\nu}\epsilon_4^\mu - 4if_{D^*D^*V}(q^\mu\epsilon_{4\nu} - g_\nu^\mu q\cdot\epsilon_4)]\\
&\times I_2^{(i)}\Big[ig_{2,2}\Theta^{\kappa\xi} + ig_{1,0}(g^{\kappa\xi} - \frac{p_1^\kappa p_1^\xi}{m_{D_1}^2})\Big]\Big[-g_\xi^\nu + \frac{q_\xi q^\nu}{m_i^2}\Big]\\
&\times\Big[-g_\kappa^\sigma + \frac{p_1^\sigma p_{1\kappa}}{m_{D_1}^2}\Big]\Big[-g_{\sigma\mu} + \frac{p_{2\sigma}p_{2\mu}}{m_{D^*}^2}\Big]\frac{\mathcal{F}^2(q^2,m_i^2)}{q^2-m_i^2}.
\end{aligned}\tag{14}$$

C. $Z^+(4430) \to D^{*+}\bar{D}_1{}^0(\bar{D}^{*0}D_1{}^+) \to D^{*+}\bar{D}^{*0}$

Fig. 2 (d) corresponds to the process $Z^+(4430) \to \bar{D}_1{}^0(p_1,\epsilon_1)D^{*+}(p_2,\epsilon_2) \to \bar{D}^{*0}(p_3,\epsilon_3)D^{*+}(p_4,\epsilon_4)$. The absorptive part of the amplitude from the π exchange is

$$\begin{aligned}
&\texttt{Abs}^{\pi}[Z^+ \to D^{*+}\bar{D}_1{}^0 \to D^{*+}\bar{D}^{*0}]\\
=& \frac{|\mathbf{p}|}{32\pi^2 M_Z}\int d\Omega\ i[ig_{Z^+D_1D^*}]\\
&\times[-\frac{ig_{D^*D^*P}}{2\sqrt{2}}\varepsilon_{\mu\tau\alpha\xi}(iq^\tau)(ip_2^\alpha + ip_4^\alpha)\epsilon_4^\xi]ig_{D_1D^*\pi}\\
&\times[-3q^\beta q^\nu + g^{\beta\nu}q^2 - \frac{g^{\nu\beta}}{m_{D^*}m_{D_1}}(p_3\cdot q)(p_1\cdot q)]\epsilon_{3\beta}\\
&\times\Big[-g_{\nu\rho} + \frac{p_{1\nu}p_{1\rho}}{m_{D_1}^2}\Big]\Big[-g^{\mu\rho} + \frac{p_2^\rho p_2^\mu}{m_{D^*}^2}\Big]\frac{\mathcal{F}^2(q^2,m_i^2)}{q^2-m_\pi^2}.
\end{aligned}\tag{15}$$

For the ρ meson exchange, the amplitude is

$$\begin{aligned}
&\texttt{Abs}^{\rho(\omega)}[Z^+ \to D^{*+}\bar{D}_1{}^0 \to D^{*+}\bar{D}^{*0}]\\
=& \frac{|\mathbf{p}|}{32\pi^2 M_Z}\int d\Omega\ i[ig_{Z^+D_1D^*}]\\
&\times I_1^{(i)}[-2ig_{D^*D^*V}p_{2\nu}\epsilon_4^\mu - 4if_{D^*D^*V}(q^\mu\epsilon_{4\nu} - g_\nu^\mu q\cdot\epsilon_4)]\\
&\times I_2^{(i)}[i\varepsilon_{\tau\sigma\xi\kappa}\frac{p_1^\kappa}{m_{D_1}}\epsilon_3^\xi\Theta_\eta^\sigma(g'_{2,2}+g'_{1,2})\\
&+i\varepsilon_{\tau\sigma\eta\kappa}\frac{p_1^\kappa}{m_{D_1}}\epsilon_{3\gamma}\Theta^{\sigma\gamma}(g'_{2,2}-g'_{1,2}) + i\varepsilon_{\tau\eta\xi\kappa}\epsilon_3^\xi\frac{p_1^\kappa}{m_{D_1}}g'_{1,0}]\\
&\times\Big[-g^{\nu\tau} + \frac{q^\tau q^\nu}{m_i^2}\Big]\Big[-g^{\sigma\eta} + \frac{p_1^\sigma p_1^\eta}{m_{D_1}^2}\Big]\Big[-g_{\sigma\mu} + \frac{p_{2\sigma}p_{2\mu}}{m_{D^*}^2}\Big]\\
&\times\frac{\mathcal{F}^2(q^2,m_i^2)}{q^2-m_i^2}.
\end{aligned}\tag{16}$$

Throughout this section, $\mathcal{F}(q^2,m_i^2)$ is the form factor to describe the structure effect in the vertex of the rescattering process $\bar{D}_1D^*(D_1\bar{D}^*) \to D^{(*)}\bar{D}^{(*)}$. We use the monopole form [17, 22]

$$\mathcal{F}(q^2,m_i^2) = \Big(\frac{m_i^2-\Lambda^2}{q^2-\Lambda^2}\Big)\tag{17}$$

where $\Lambda = m_i + \alpha\Lambda_{QCD}$ with $\Lambda_{QCD} = 220$ MeV and $\alpha = 1 \sim 3$.

IV. RESULTS AND DISCUSSION

We collect the values of these coupling constants below

$$\begin{aligned}
g_{D^*D^*P} &= \frac{g_{D^*DP}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_\pi},\\
f_{D^*DV} &= \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}},\ \ g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}},\\
g_{D_1D^*\pi} &= -\frac{\sqrt{m_{D^*}m_{D_1}}}{3f_\pi\Lambda_\chi}(h_1+h_2),\ \ g_V = \frac{m_\rho}{f_\pi},
\end{aligned}$$

where $g = 0.59$, $\lambda = 0.56$ GeV^{-1}, $\beta = 0.9$ are the parameters in the effective Lagrangian [20, 22, 23, 24]. Casalbuoni and his collaborators extracted $h' = (h_1+h_2)/\Lambda_\chi = 0.55$ GeV^{-1} with the available experimental data [20]. In Ref. [21], Zhu and Dai gave $g_{2,2} = \sqrt{6}g_d/6$, $g_{1,0} = -\sqrt{6}g_s/3$, $g'_{2,2} = \sqrt{6}g_d/4$, $g'_{1,2} = \sqrt{6}g_d/12$, $g'_{1,0} = \sqrt{6}g_s/6$, $g_s = 2.1$ and $g_d = 3.8$ GeV^{-2} in the framework of QCD sum rule.

For the parameter η, we use $\eta = 0.4$ GeV^{-2} [25]. The meson masses are from PDG: $m_{D^0} = 1864.5$ MeV, $m_{D^+} = 1869.3$ MeV. $m_{D_1} = 2422.3$ MeV, $m_{D^{*0}} = 2006.7$ MeV, $m_{D^{*+}} = 2010.0$ MeV, $m_\pi = 135.0$ MeV, $m_\rho = 775.5$ MeV, $m_\omega = 782.7$ MeV [26].

With the above parameters, we plot the dependence of the branching ratio of the two-body open charm decays $Z^+(4430) \to D^+\bar{D}^{*0}, D^{*+}\bar{D}^0, D^{*+}\bar{D}^{*0}$ on α in Fig. 3. In Table I, we also list show the branching ratios of $D^+\bar{D}^{*0}$, $D^{*+}\bar{D}^0$, $D^{*+}\bar{D}^{*0}$ modes with typical values of α.

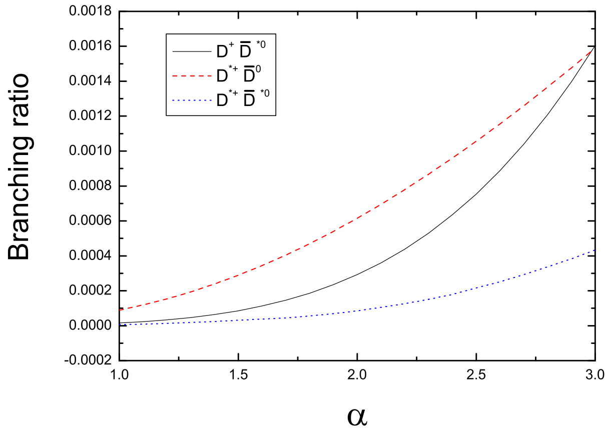


FIG. 3: The dependence of the branching ratios on α. The solid, dashed and dotted lines correspond to the $D^+\bar{D}^{*0}$, $D^{*+}\bar{D}^0$ and $D^{*+}\bar{D}^{*0}$ modes respectively.

In short summary, we have calculated the two-body open charm decay widths assuming $Z^+(4430)$ is a pseudoscalar molecular state. Our numerical results show that the branching ratios of the two-body decays $Z^+(4430) \to D^+\bar{D}^{*0}, D^{*+}\bar{D}^0 D^{*+}\bar{D}^{*0}$ are around $10^{-5} \sim 10^{-3}$, which is strongly suppressed compared to the main decay modes $D^*\bar{D}^*\pi$ if it's a molecular state with $J^P = 0^-$. However, they are only moderately suppressed compared with the discovery mode $\psi'\pi^+$.

As an excellent candidate of the exotic states, the observation of $Z^+(4430)$ has stimulated several speculations about its structure. The tetraquark scheme seems an interesting choice. However, there is no reliable dynamics forbidding the quarks and anti-quarks to regroup and fall apart. With plentiful phase space $Z^+(4430)$ will easily fall apart into final states such $\psi\pi$, $\psi'\pi$, $\psi(1D)\pi$, $\eta_c\rho$, $\chi_{cJ}\rho$, $D\bar{D}^*$ etc if $Z^+(4430)$ is a tetraquark. Especially the two-body open charm modes $D^+\bar{D}^{*0}, D^{*+}\bar{D}^0, D^{*+}\bar{D}^{*0}$ etc will become one of the main decay modes, in sharp contrast with the case of $Z^+(4430)$ being a molecular state. Thus these modes provide a smoking gun to distinguish the molecular and tetraquark schemes. We strongly urge experimentalists to look for these two-body open charm decay modes together with the hidden charm mode $\chi_{cJ}\rho^+$.

TABLE I: The branching ratios with typical values of α.

α	1.0	1.4	1.8	2.2	2.6	3.0
$B[Z^+ \to D^+\bar{D}^{*0}]$	1.6×10^{-5}	6.4×10^{-5}	1.9×10^{-4}	4.4×10^{-4}	8.9×10^{-4}	1.6×10^{-3}
$B[Z^+ \to D^{*+}\bar{D}^0]$	8.9×10^{-5}	2.4×10^{-4}	4.7×10^{-4}	7.8×10^{-4}	1.2×10^{-3}	1.6×10^{-3}
$B[Z^+ \to D^{*+}\bar{D}^{*0}]$	6.3×10^{-6}	2.4×10^{-5}	5.6×10^{-5}	1.3×10^{-4}	2.5×10^{-4}	4.3×10^{-4}

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